

AD-A163 387

INITIATOR DURATION EFFECT ON PULSED CHAIN REACTION  
CHEMICAL LASER SCALING LAWS(U) AEROSPACE CORP EL  
SEGUNDO CA H MIRELS 15 NOV 85 TR-0886(6930-01)-3

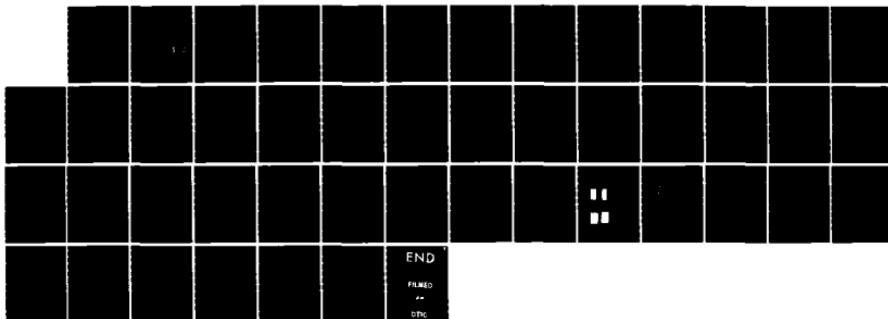
UNCLASSIFIED

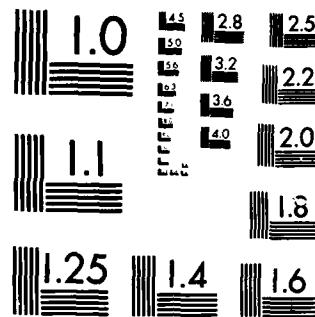
SD-TR-85-82 F04701-85-C-0886

1/1

F/G 20/5

NL





MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

12

AD-A163 387

Initiator Duration Effect on Pulsed  
Chain Reaction Chemical Laser  
Scaling Laws

HAROLD MIRELS  
Aerophysics Laboratory  
Laboratory Operations  
The Aerospace Corporation  
El Segundo, Calif. 90245

15 November 1985

DTIC  
SELECTED  
JAN 23 1986  
S D

APPROVED FOR PUBLIC RELEASE;  
DISTRIBUTION UNLIMITED

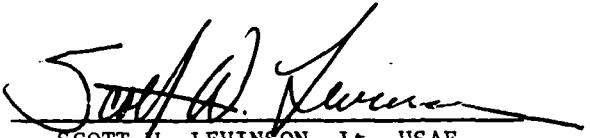
MS. FILE COPY

Prepared for  
SPACE DIVISION  
AIR FORCE SYSTEMS COMMAND  
Los Angeles Air Force Station  
P.O. Box 92960, Worldway Postal Center  
Los Angeles, CA 90009-2960

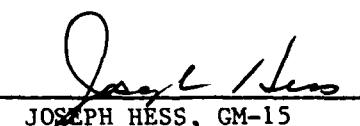
This report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract No. F04701-85-C-0086 with the Space Division, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009-2960. It was reviewed and approved for The Aerospace Corporation by W. P. Thompson, Director, Aerophysics Laboratory. Lieutenant Scott W. Levinson, SD/YNS, was the project officer for the Mission-Oriented Investigation and Experimentation (MOIE) Program.

This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.



SCOTT W. LEVINSON, Lt., USAF  
MOIE Project Officer  
SD/YNS



JOSEPH HESS, GM-15  
Director, AFSTC West Coast Office  
AFSTC/WCO OL-AB

## UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SD-TR-85-82	2. GOVT ACCESSION NO. AD A163387	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) INITIATOR DURATION EFFECT ON PULSED CHAIN REACTION CHEMICAL LASER SCALING LAWS	5. TYPE OF REPORT & PERIOD COVERED	
7. AUTHOR(s) Harold Mirels	6. PERFORMING ORG. REPORT NUMBER TR-0086(6930-01)-3	
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Aerospace Corporation El Segundo, Calif. 90245	8. CONTRACT OR GRANT NUMBER(s) F04701-85-C-0086	
11. CONTROLLING OFFICE NAME AND ADDRESS Space Division Los Angeles Air Force Station Los Angeles, Calif. 90009-2960	12. REPORT DATE 15 November 1985	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	13. NUMBER OF PAGES 41	
	15. SECURITY CLASS. (of this report) Unclassified	
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Chemical Laser Pulsed Chemical Laser Scaling Laws		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  The effect of initiator pulse duration on the performance of a pulsed chain reaction chemical laser is investigated using a two-level vibrational model. Analytic results are presented for a saturated laser in the limits of weak and strong initiation. The initiator is assumed to provide a uniform (e.g., electron beam), parabolic (e.g., flash lamp), or power law variation of F-atom production rate $F$ with time. Laser performance is presented as a function of $t_B/t_e$ where $t_B$ and $t_e$ are initiator and laser pulse times, respectively. In		

DD FORM 1473  
[REDACTED] IFACSIMILE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

19. KEY WORDS (Continued)

20. ABSTRACT (Continued)

the weak initiation regime, an increase of  $t_B/t_e$  from zero to one results in a decrement in laser output energy of 20 and 33-1/3% for a flash lamp and electron-beam initiator, respectively. In the strong initiation regime, an increase of  $t_B/t_e$  from zero to one results in an energy decrement of only 5 and 10% for a flash lamp and electron-beam initiator, respectively. In each case, the laser pulse time  $t_e$  is increased by a factor of two as  $t_B/t_e$  increases from zero to one.

Keywords: Scaling Laws, Pulse-Width, Lasers.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

PREFACE

The author is indebted to R. Hofland and S. T. Amimoto, whose experimental work and discussions stimulated the present study. The author also acknowledges K. L. Foster for numerical support.

Accession For	
NTIS	CRA&I
DTIC	TAB
Unannounced	
Justification	
By _____	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

## CONTENTS

PREFACE .....	1
I. INTRODUCTION.....	7
II. THEORY.....	9
A. Laser Performance.....	9
B. Chain Reaction.....	10
C. Uniform Value of $F_B$ (Electron Beam).....	14
D. Parabolic Variation of $F_B$ (Flash Lamp).....	19
E. Power Law Variation of $F_B$ .....	22
III. DISCUSSION.....	27
A. Physical Significance of Parameters.....	27
B. Initiator Pulse Length Effect.....	30
C. Laser Pulse Shape.....	32
D. Region of Validity.....	35
E. Improved Scaling Laws.....	35
IV. CONCLUDING REMARKS.....	39
REFERENCES .....	41
APPENDIXES .....	43
A. TYPICAL VALUES OF PARAMETERS.....	43
B. COMPARISON WITH REFERENCE 8.....	45
SYMBOLS.....	47

## FIGURES

1. Characteristic Times for Electron-Beam Initiated Pulsed Chain Reaction Chemical Laser,  $t_B/t_e = 0(1)$ ,  $t_t/t_e \ll 1$ ,  $t_e/t_p \ll 1$ ..... 8
2. F-Atom Production Rate and Concentration Resulting from Idealized Flash and Electron Beam..... 15
3. F-Atom Concentration Associated with Power Law Initiation,  $F_B/F_{BT} = \zeta^n$  ..... 23
4. Laser Output Intensity Variation for Weak Initiation,  $(\phi/B)^2 \ll 1$ ..... 33
5. Laser Output Intensity Variation for Strong Initiation,  $(B/\phi) \ll 1$ .... 34
6. Effect of Electron-Beam Duration on Variation of Laser Output Intensity with Time in Strong Initiation Regime..... 36
7. Variation of Laser Intensity with Time for Flash-Initiated Laser in Strong Initiation Regime..... 37

## I. INTRODUCTION

Pulsed chain reaction HF(DF) chemical lasers are generally initiated by either a flash lamp<sup>1-3</sup> or an electron-beam discharge.<sup>4-7</sup> The discharge generates fluorine atoms that initiate the H<sub>2</sub>-F<sub>2</sub> (D<sub>2</sub>-F<sub>2</sub>) chain reaction. Scaling laws for the case where the flash lamp or electron-beam discharge time  $t_B$  is short, compared with the laser pulse time  $t_e$ , have been presented.<sup>8</sup> However, flash lamp discharge times are generally of the order of the laser pulse time (e.g., Reference 3). Although electron-beam discharge times can be short, compared with the laser pulse time, the effect of making these times of the same order is currently being investigated experimentally.<sup>9</sup> Hence, it is of interest to determine the effect of the parameter  $t_B/t_e$  on pulsed chain reaction chemical laser scaling laws. Such a study is pursued herein.

A pulsed chain reaction HF(DF) chemical laser is considered. The solution in Reference 8, which is applicable in the limit  $t_B/t_e \rightarrow 0$ , is generalized to consider  $t_B/t_e = 0$  (1). During the course of the present development, it is assumed that the laser is saturated and that the following inequalities apply

$$t_t/t_e \ll 1, \quad t_e/t_p \ll 1 \quad (1)$$

where  $t_t$  and  $t_p$  denote an initial transient and the duration, respectively, of the chain (pumping) reaction. These quantities are illustrated in Fig. 1. Results are presented for a uniform (electron beam) and for a parabolic (flash lamp) variation of fluorine atom production rate by the reaction initiating discharge. The effect of a power law variation, with time, is also deduced. Symbols are defined at the end of this report. Typical values of initial reactant concentrations, rate coefficients, and reaction times are noted in Appendix A. These values confirm that the inequalities in Eq. (1) are generally satisfied.

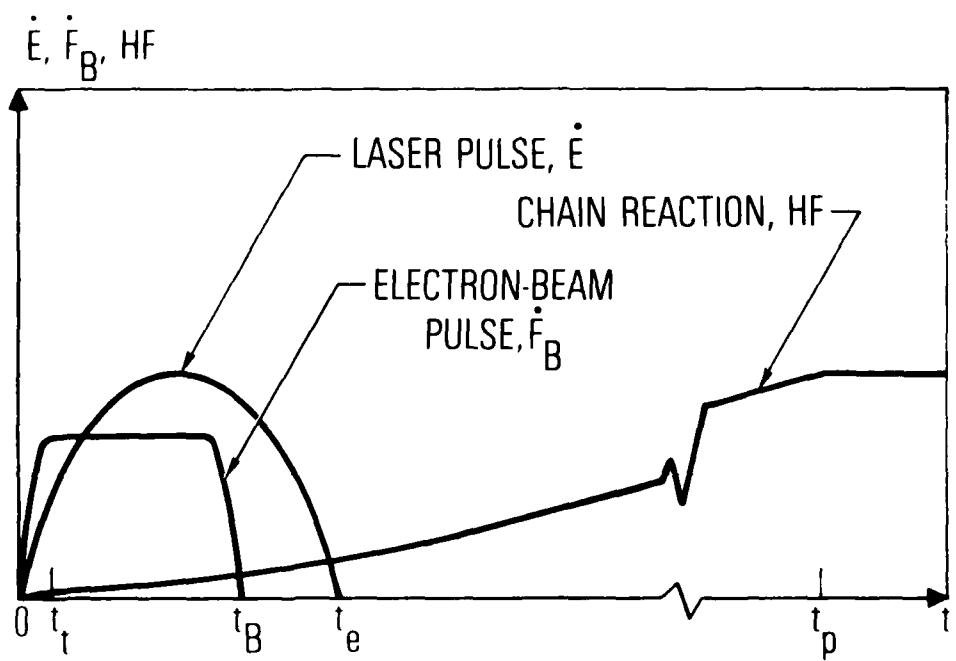


Fig. 1. Characteristic Times for Electron-Beam Initiated Pulsed Chain Reaction Chemical Laser,  $t_B/t_e = 0(1)$ ,  $t_l/t_e \ll 1$ ,  $t_e/t_p \ll 1$

## II. THEORY

A general solution is presented for pulsed HF chain reaction chemical laser performance. A saturated laser is assumed. Results are then obtained for a uniform, a parabolic, and a power law variation, with time, of F-atom production rate.

### A. LASER PERFORMANCE

Consider a two-level model



where  $HF_u$ ,  $HF_l$ , and  $HF$  denote upper level, lower level, and net concentration of HF, respectively. Small signal gain is obtained from

$$g = \sigma(HF_u - HF_l) \quad (2b)$$

The net concentration of HF is determined only by the chain (pump) reaction, which is discussed later and which is assumed to create HF in the upper level only. Thus

$$\left(\frac{dHF}{dt}\right)_p = \left(\frac{dHF_u}{dt}\right)_p \equiv \dot{HF} \quad (3)$$

The net rate of change of  $HF_u$  is then

$$\frac{dHF_u}{dt} = \dot{HF} - (k_{cd}^{HF} HF + \sum_i k_{cd}^{M_i} M_i) HF_u - \frac{gI}{\epsilon} \quad (4)$$

where the first, second, and third terms on the right-hand side of Eq. (4) denote pumping, collisional deactivation, and stimulated emission and absorption terms, respectively. The quantity  $k_{cd}^{HF}$  is the rate coefficient for collisional deactivation of  $HF_u$  by HF and is later shown to be the dominant

collisional deactivation process for efficient-pulsed chemical lasers. Other collision partners ( $H_2$ ,  $F_2$ , He, F, and H) are denoted by  $M_i$  in Eq. (4). Eq. (4) can be expressed in terms of  $g$  and HF by noting  $2HF_u = (g/\sigma) + HF$ . The result is

$$\frac{d(g/\sigma)}{dt} = HF - (k_{cd}^{HF} HF + \sum_i^{M_i} k_{cd}^{HF} M_i) \left( \frac{g}{\sigma} + HF \right) - \frac{2gI}{\epsilon} \quad (5)$$

We consider a saturated laser, namely

$$g \rightarrow 0, \quad I \rightarrow \infty, \quad gI \equiv E = \text{finite} \quad (6)$$

where  $gI = E$  is the rate of photon energy release per unit volume. Eq. (5) becomes

$$\frac{d}{dt} \frac{2E}{\epsilon H_{2,0}} = \frac{HF}{H_{2,0}} - k_{cd}^{HF} H_{2,0} \left[ \left( \frac{HF}{H_{2,0}} \right)^2 + 2B \frac{HF}{H_{2,0}} \right] \quad (7a)$$

$$\frac{2E}{\epsilon H_{2,0}} = \int_0^t \frac{2E}{\epsilon H_{2,0}} dt = \frac{HF}{H_{2,0}} - k_{cd}^{HF} H_{2,0} \int_0^t \left[ \left( \frac{HF}{H_{2,0}} \right)^2 + 2B \frac{HF}{H_{2,0}} \right] dt \quad (7b)$$

where  $H_{2,0}$  is the initial value of  $H_2$  and

$$B = \left( \sum_i^{M_i} k_{cd}^{HF} M_i \right) / (2H_{2,0} k_{cd}^{HF}) \quad (7c)$$

Eqs. (7a, b, and c) define laser performance, provided HF is a known function of time. The latter is evaluated in the following section.

#### B. CHAIN REACTION

HF is created by the chain reaction



where subscripts c and h denote the cold and hot reaction, respectively. The F-atom production per unit volume per unit time, due directly to flash or electron-beam irradiation, is denoted by  $\dot{F}_B$ . It follows that

$$\dot{F}_B \equiv dF_B/dt \quad (9a)$$

$$F_B = \int_0^t \dot{F}_B dt \quad (9b)$$

$$F_{B,T} = \int_0^{t_B} \dot{F}_B dt \quad (9c)$$

where  $F_{B,T}$  denotes the total number of F-atoms, per unit volume, generated directly by the discharge. Rate equations corresponding to Eq. (8) are

$$dF/dt = k_h F_2 H - k_c H_2 F + \dot{F}_B \quad (10a)$$

$$dH/dt = k_c H_2 F - k_h F_2 H \quad (10b)$$

$$d(HF)_c/dt = k_c H_2 F = - dH_2/dt \quad (10c)$$

$$d(HF)_h/dt = k_h F_2 H = - dF_2/dt \quad (10d)$$

$$dHF/dt = k_c H_2 F + k_h F_2 H \quad (10e)$$

with boundary conditions, at  $t = 0$ , given by

$$F = H = 0 \quad F_2 = F_{2,0} \quad H_2 = H_{2,0} \quad (11)$$

Eqs. (10a) and (10b) indicate

$$F + H = F_B \quad (12)$$

Thus, the total number of F and H atoms, at any instant, equals the total number of F-atoms generated up to that instant by the discharge.

It will be shown that the laser pulse time,  $t_e$ , is short compared with the time required for the chain reaction to go to completion,  $t_p$ . In the regime of interest (i.e.,  $t/t_e \ll 1$ )  $F_2$  and  $H_2$ , in Eq. (10a), can then be approximated by their initial values, namely

$$F_2/F_{2,0} = H_2/H_{2,0} = 1 + 0 (t_e/t_p) \quad (13)$$

Eqs. (10a) and (12) become

$$\frac{dF}{dt} + \frac{F}{t_t} = k_h F_{2,0} F_B + \dot{F}_B \quad (14a)$$

where

$$t_t = (k_c H_{2,0} + k_h F_{2,0})^{-1} \quad (14b)$$

The integral of Eq. (14) can be expressed as

$$F = RF_B + (1 - R)e^{-t/t_t} \int_0^t e^{t/t_t} F_B dt \quad (15a)$$

where

$$R \equiv t_t k_h F_{2,0} = k_h F_{2,0} / (k_c H_{2,0} + k_h F_{2,0}) \quad (15b)$$

Note that  $R$  is less than one. The quantity  $t_t$  characterizes an initial transient and is small, in general, compared with the laser pulse time  $t_e$  (e.g., Appendix A). Use of a mean value for  $F_B$ , in Eq. 15(a), indicates that for times of the order of  $t_e$

$$F = RF_B [1 + O(t_t/t_e)] \quad (16)$$

The present solution for  $F$  is similar to the "steady state" solution in Reference 8. The quantity  $F_B$  in Eq. (16) replaces  $F_{B,T}$  in the equivalent expression, Eq. (16), in Reference 8. The present solution may be viewed as a quasi steady-state solution in that if, at any instant, the initiator pulse is terminated, the  $F$  and  $H$  atom concentrations remain constant with time thereafter. The corresponding values of  $F$  and  $H$  are given by Eqs. (16) and (12), respectively. Substitution of Eqs. (12), (13), and (16) into Eq. (10e), and integration, provides the variation of  $HF$  with time, namely

$$\frac{HF}{2 H_{2,0}} = \int_0^t \frac{F_B}{F_{B,T}} \frac{dt}{t_p} \quad (17)$$

where

$$t_p \equiv (k_c R F_{B,T})^{-1} \quad (18a)$$

characterizes the time for the chain reaction to go to completion. We introduce, for future reference, the related parameter<sup>8</sup>

$$\phi^2 \equiv \frac{1}{2 H_{2,0} k_{cd}^{HF} t_p} = \frac{k_h}{2 k_{cd}^{HF}} \frac{(F_{B,T}/F_{2,0})(F_2/H_2)_0^2}{1 + (k_h/k_c)(F_2/H_2)_0} \quad (18b)$$

and observe the identity

$$B/(t_p \phi^2) = \sum_i k_{cd} M_i M_i \quad (18c)$$

The quantity  $\phi^2$  is a measure of the initiation strength,  $F_{B,T}/F_{2,0}$ . Typical values of these parameters are noted in Appendix A.

Eqs. (7) and (17) define pulsed chain reaction chemical laser performance corresponding to a specified discharge-induced temporal variation of  $F_B$ . Numerical results for uniform, parabolic, and power law variations of  $F_B$ , with time, are deduced in the following sections.

#### C. UNIFORM VALUE OF $F_B$ (ELECTRON BEAM)

The rate of F-atom production by the electron beam,  $F_B$ , is assumed to be a constant during the electron-beam discharge time,  $t_B$  (Fig. 2). The case of an arbitrary magnitude of  $F_B$  is first considered. Limiting solutions corresponding to relatively small and relatively large magnitudes of  $F_B$  are then noted.

##### 1. ARBITRARY INITIATION LEVEL

The concentration of discharge-produced F-atoms, at any instant, is (Fig. 2)

$$F_B/F_{B,T} = \zeta \quad \zeta < 1 \quad (19a)$$

$$= 1 \quad \zeta > 1 \quad (19b)$$

where  $\zeta = t/t_B$ : The resulting concentration of HF is, from Eq. (17),

$$\frac{1}{2} \frac{t_p}{t_B} \frac{HF}{H_{2,0}} = \frac{\zeta^2}{2} \quad \zeta < 1 \quad (20a)$$

$$= \zeta - \frac{1}{2} \quad \zeta > 1 \quad (20b)$$

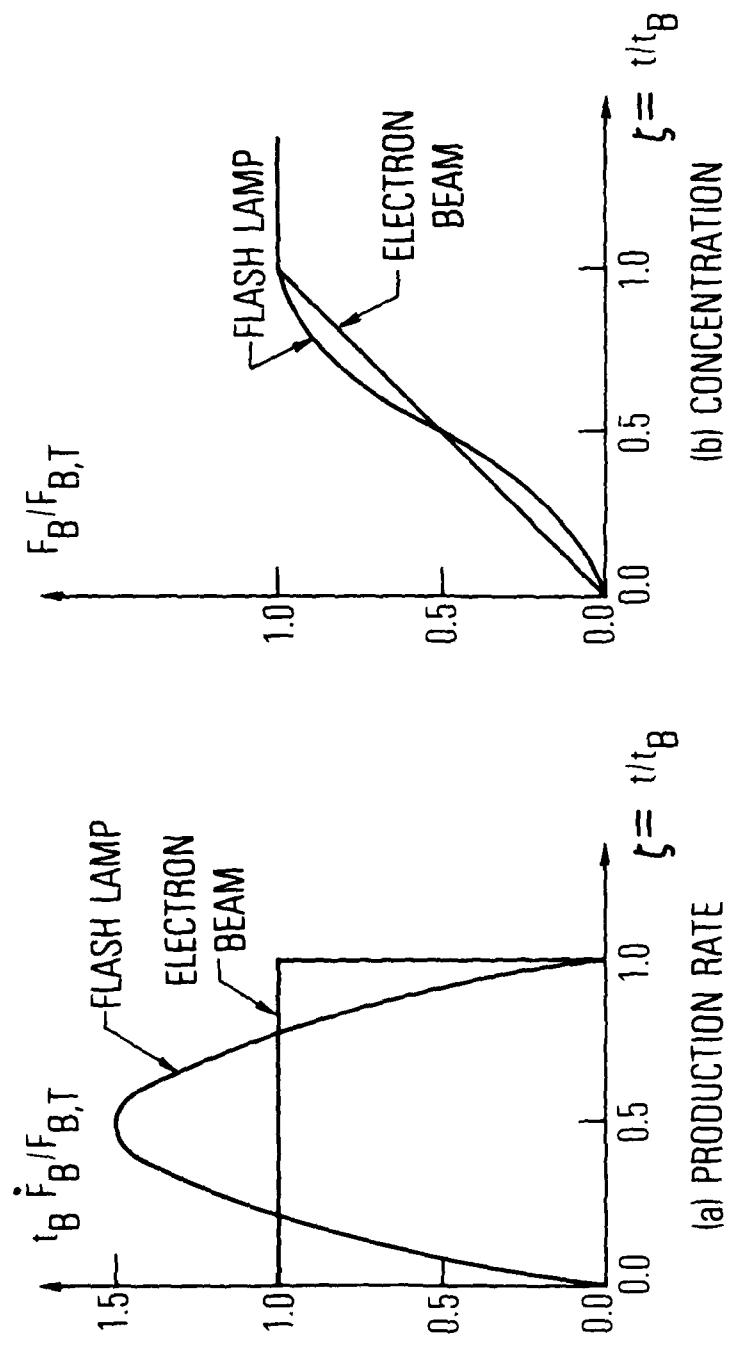


FIG. 2. F-Atom Production Rate and Concentration Resulting from Idealized Flash and Electron Beam. (a) Production rate and (b) concentration

Substitution of Eq. (20) into Eq. (7) provides the variation of laser intensity  $E$  and output  $E$  as a function of time. The result for laser intensity is

$$\frac{t_p}{\epsilon H_{2,0}} \frac{E}{\epsilon} = \zeta - \left( \frac{1}{2\phi} \frac{t_B}{t_p} \zeta^2 \right)^2 \left( 1 + 2 \frac{t_p}{t_B} \frac{B}{\zeta^2} \right) \quad \zeta < 1 \quad (21a)$$

$$= 1 - \left[ \frac{1}{2\phi} \frac{t_B}{t_p} (2\zeta - 1) \right]^2 \left( 1 + 2 \frac{t_p}{t_B} \frac{B}{2\zeta - 1} \right) \quad \zeta > 1 \quad (21b)$$

where  $\phi$  has been defined in Eq. (18). The net energy liberated, up to time  $\zeta$ , is

$$\frac{t_p}{t_B} \frac{E}{\epsilon H_{2,0}} = \frac{\zeta^2}{2} - \left( \frac{1}{2\phi} \frac{t_B}{t_p} \right)^2 \left( 1 + \frac{10}{3} \frac{t_p}{t_B} \frac{B}{\zeta^2} \right) \frac{\zeta^5}{5} \quad \zeta < 1 \quad (22a)$$

$$= \zeta - \frac{1}{2} - \left( \frac{1}{2\phi} \frac{t_B}{t_p} \right)^2 \left[ \frac{5(2\zeta - 1)^3 + 1}{30} + \frac{t_p}{t_B} \frac{B}{6} \frac{3(2\zeta - 1)^2 + 1}{6} \right] \quad \zeta < 1 \quad (22b)$$

The laser pulse length  $\zeta_e$  is determined by finding the value of  $\zeta$ , other than  $\zeta = 0$ , for which  $E = 0$  in Eq. (21). The dependence of  $\zeta_e$  on  $\phi t_p / t_B$  and  $\phi / B$  is found from

$$\zeta_e = (2\phi \frac{t_p}{t_B})^{2/3} \left( 1 + \frac{2Bt_p}{t_B \zeta_e^2} \right)^{-1/3} \quad \zeta_e < 1 \quad (23a)$$

$$\zeta_e = \frac{1}{2} + \frac{\phi t_p}{2 t_B} \left\{ \left[ 4 + \left( \frac{B}{\phi} \right)^2 \right]^{1/2} - \frac{B}{\phi} \right\} \quad \zeta_e > 1 \quad (23b)$$

The corresponding net laser output energy  $E_e$  is determined from Eqs. (22) and (23). The case  $\zeta_e > 1$  is of primary interest since this corresponds to situations where the discharge time  $t_B$  is less than or equal to the laser pulse time  $t_e$ .

Eqs. (20) to (23) provide a general solution for electron-beam-initiated chemical lasers. The dependent variables are functions of  $\zeta$  and the parameters  $\phi t_p / t_B$  and  $B/\phi$ . It is convenient to consider the solution in the limits  $(\phi/B)^2 \ll 1$  and  $(B/\phi) \ll 1$ . These limits are termed weak and strong initiation,<sup>8</sup> respectively, and are obtained herein.

## 2. WEAK INITIATION $(\phi/B)^2 \ll 1$

The evaluation of Eqs. (21) to (23) in the present limit yields

$$\zeta_e = \frac{2\phi^2 t_p}{B t_B} \left[ 1 + O\left(\frac{\phi \zeta_e^{1/2}}{B}\right)^2 \right] \quad \zeta_e < 1 \quad (24a)$$

$$= \frac{1}{2} + \frac{\phi^2 t_p}{B t_B} \left[ 1 + O\left(\frac{\phi}{B}\right)^2 \right] \quad \zeta_e > 1 \quad (24b)$$

$$\frac{B}{\phi^2} \frac{E_e}{\epsilon H_{2,0}} = \frac{\zeta_e}{3} \left[ 1 + O\left(\frac{\phi \zeta_e^{1/2}}{B}\right)^2 \right] \quad \zeta_e < 1 \quad (25a)$$

$$= \frac{1}{2} \left[ 1 - \frac{1}{3} \frac{1}{(2\zeta_e - 1)^2} \right] \left[ 1 + O\left(\frac{\phi}{B}\right)^2 \right] \quad \zeta_e > 1 \quad (25b)$$

$$\frac{t_p E}{\epsilon H_{2,0}} = \zeta \left( 1 - \frac{\zeta}{\zeta_e} \right) \quad \zeta_e < 1 \quad (26)$$

$$= \zeta \left( 1 - \frac{\zeta}{2\zeta_e - 1} \right) \quad \zeta < 1, \zeta_e > 1 \quad (27a)$$

$$= 1 - \frac{2\zeta - 1}{2\zeta_e - 1} \quad \zeta > 1, \zeta_e > 1 \quad (27b)$$

Eqs. (24) to (27) provide the laser pulse length  $\zeta_e$ , the net output energy  $E_e$ , and the variation of laser intensity  $E$  with time  $\zeta$ . For convenience, the error terms are not explicitly displayed in Eqs. (26) and (27).

When  $\zeta_e < 1$ , the electron beam continues to create F-atoms after the lasing pulse is over. The latter play no role in the lasing process. For  $\zeta_e < 1$ , the F-atoms created during the actual lasing process equal  $F_{B,e} = \zeta_e F_{B,T}$ . As a result, the parameters  $\phi^2$  and  $t_B/t_p$  appear in the form  $\zeta_e \phi^2$  and  $\zeta_e t_B/t_p$  in Eqs. (24a) and (25a). That is, laser performance depends only on the F-atoms created during the lasing process, as expected.

Additional discussion of these results is provided in Section III.

### 3. STRONG INITIATION ( $B/\phi$ ) $\ll 1$

Eqs. (21) to (23) become, in the present limit,

$$\zeta_e = \left( \frac{2\phi t_p}{t_B} \right)^{2/3} \left[ 1 + O\left( \frac{B}{\phi \zeta_e^{1/2}} \right) \right] \quad \zeta_e < 1 \quad (28a)$$

$$= \frac{1}{2} + \frac{\phi t_p}{t_B} \left[ 1 + O\left( \frac{B}{\phi} \right) \right] \quad \zeta_e > 1 \quad (28b)$$

$$\frac{1}{\phi} \frac{E_e}{\epsilon H_{2,0}} = \frac{3}{5} \zeta_e^{1/2} \left[ 1 + O\left( \frac{B}{\phi \zeta_e^{1/2}} \right) \right] \quad \zeta_e < 1 \quad (29a)$$

$$= \frac{2}{3} \left[ 1 - \frac{1}{10} \frac{1}{(2\zeta_e - 1)^3} \right] \left[ 1 + O\left( \frac{B}{\phi} \right) \right] \quad \zeta_e > 1 \quad (29b)$$

$$\frac{t_p E}{\epsilon H_{2,0}} = \zeta \left[ 1 - \left( \frac{\zeta}{\zeta_e} \right)^3 \right] \quad \zeta_e < 1 \quad (30)$$

$$= \zeta \left[ 1 - \frac{\zeta^3}{(2\zeta_e - 1)^2} \right] \quad \zeta < 1, \zeta_e > 1 \quad (31a)$$

$$= 1 - \left( \frac{2\zeta - 1}{2\zeta_e - 1} \right)^2 \quad \zeta > 1, \zeta_e > 1 \quad (31b)$$

where the error terms are again omitted from Eqs. (30) and (31).

Dependent variables in Eqs. (28a) and (29a) depend on  $\phi^2 \zeta_e$  and  $(t_B/t_p)\zeta_e$  for reasons discussed previously.

#### D. PARABOLIC VARIATION OF $F_B$ (FLASH LAMP)

The quantity  $F_B$  is now assumed to have a parabolic variation with time (Fig. 2). The parabolic variation approximates F-atom production by a flash lamp.<sup>3</sup> The case of an arbitrary initiation level is treated. Limiting solutions for weak and strong initiation are then noted.

##### 1. ARBITRARY INITIATION LEVEL

The concentration of F-atoms produced by the flash lamp is

$$t_B \dot{F}_B / F_{B,T} = 6\zeta(1 - \zeta) \quad \zeta < 1 \quad (32a)$$

$$= 0 \quad \zeta > 1 \quad (32b)$$

$$F_B / F_{B,T} = 3\zeta^2 [1 - (2/3)\zeta] \quad \zeta < 1 \quad (32c)$$

$$= 1 \quad \zeta > 1 \quad (32d)$$

The resulting HF concentration is

$$\frac{1}{2} \frac{t_p}{t_B} \frac{HF}{H_{2,0}} = \zeta^3 \left( 1 - \frac{\zeta}{2} \right) \quad \zeta < 1 \quad (33a)$$

$$= \zeta - \frac{1}{2} \quad \zeta > 1 \quad (33b)$$

and the variation of laser intensity with time is

$$\frac{\dot{t}_p E}{\epsilon H_{2,0}} = 3\zeta^2 \left(1 - \frac{2}{3}\zeta\right) - \left(\frac{t_B}{\phi t_p}\right)^2 \left(\zeta^3 - \frac{\zeta^4}{2}\right) \left(\zeta^3 - \frac{\zeta^4}{2} + \frac{B t_p}{t_B}\right) \quad \zeta < 1 \quad (34a)$$

$$= 1 - \left(\frac{t_B}{\phi t_p}\right)^2 \left(\zeta - \frac{1}{2}\right) \left(\zeta - \frac{1}{2} + \frac{B t_p}{t_B}\right) \quad \zeta > 1 \quad (34b)$$

Integration of Eq. (34) provides the net energy output up to time  $\zeta$ ,

$$\frac{t_p}{t_B} \frac{E}{\epsilon H_{2,0}} = \zeta^3 - \frac{\zeta^4}{2} - \left(\frac{t_B}{\phi t_p}\right)^2 \left[\frac{\zeta^7}{7} - \frac{\zeta^8}{8} + \frac{\zeta^9}{36} + \frac{B t_p}{t_B} \left(\frac{\zeta^4}{4} - \frac{\zeta^5}{10}\right)\right] \quad \zeta < 1 \quad (35a)$$

$$= \zeta - \frac{1}{2} - \left(\frac{t_B}{\phi t_p}\right)^2 \left[\frac{\zeta^3}{3} - \frac{\zeta^2}{2} + \frac{\zeta}{4} - \frac{19}{504} + \frac{B t_p}{2 t_B} \left(\zeta^2 - \zeta + \frac{3}{10}\right)\right] \quad \zeta > 1 \quad (35b)$$

The laser pulse length, corresponding to  $E = 0$  in Eq. (34), is determined from

$$\left(\frac{\phi t_p}{t_B}\right)^2 = \frac{\zeta_e(2 - \zeta_e)}{2(3 - 2\zeta_e)} \left[\zeta_e^3 - \frac{\zeta_e^4}{2} + \frac{B t_p}{t_B}\right] \quad \zeta_e < 1 \quad (36a)$$

$$\zeta_e = \frac{1}{2} + \frac{1}{2} \frac{\phi t_p}{t_B} \left\{ \left[4 + \left(\frac{B}{\phi}\right)^2\right]^{1/2} - \frac{B}{\phi} \right\} \quad \zeta_e > 1 \quad (36b)$$

Output power per pulse is obtained by evaluating Eq. (35) at values of  $\zeta_e$  defined by Eq. (36). Limiting solutions for weak and strong initiation are noted herein.

## 2. WEAK INITIATION $(\phi/B)^2 \ll 1$

In the present limit, the solutions for pulse length, energy, and intensity become, respectively

$$\frac{\phi^2 t_p}{B t_B} = \frac{\zeta_e(2 - \zeta_e)}{2(3 - 2\zeta_e)} \left[1 + 0 \left(\frac{\phi}{B} \zeta_e\right)^2\right] \quad \zeta_e < 1 \quad (37a)$$

$$= \left(\zeta_e - \frac{1}{2}\right) \left[1 + 0 \left(\frac{\phi}{B}\right)^2\right] \quad \zeta_e > 1 \quad (37b)$$

$$\frac{B}{\phi^2} \frac{E_e}{\epsilon H_{2,0}} = \frac{\zeta_e^2 (3 - 2\zeta_e) (5 - 4\zeta_e + \zeta_e^2)}{5(2 - \zeta_e)^2} \quad \zeta_e < 1 \quad (38a)$$

$$= \frac{2(5\zeta_e^2 - 5\zeta_e + 1)}{5(2\zeta_e - 1)^2} \quad \zeta_e > 1 \quad (38b)$$

$$\frac{t_p E}{\epsilon H_{2,0}} = \zeta_e^2 (3 - 2\zeta_e) \left[ \frac{3 - 2\zeta_e}{3 - 2\zeta_e} - \frac{\zeta_e (2 - \zeta_e)}{\zeta_e (2 - \zeta_e)} \right] \quad \zeta_e < 1 \quad (39a)$$

$$= \zeta_e^2 (3 - 2\zeta_e) - \frac{\zeta_e^3 (2 - \zeta_e)}{(2\zeta_e - 1)} \quad \zeta_e < 1, \zeta_e > 1 \quad (39b)$$

$$= 1 - \frac{(2\zeta_e - 1)}{(2\zeta_e - 1)} \quad \zeta_e > 1, \zeta_e > 1 \quad (39c)$$

The F-atoms generated by the flash up to time  $\zeta_e$  is, for  $\zeta_e < 1$ ,  $F_{B,e}/F_B$ ,  $T = \zeta_e^2 (3 - 2\zeta_e)$ . Hence, the parameters  $\phi^2$  and  $t_B/t_p$  appear as the products  $\phi^2 \zeta_e^2 (3 - 2\zeta_e)$  and  $(t_B/t_p) \zeta_e^2 (3 - 2\zeta_e)$  in Eqs. (37a) and (38a).

### 3. STRONG INITIATION ( $B/\phi \ll 1$ )

Corresponding results for strong initiation are

$$\frac{\phi t_p}{t_B} = \left[ \frac{\zeta_e^4}{4} \frac{(2 - \zeta_e)^2}{3 - 2\zeta_e} \right]^{1/2} \left[ 1 + 0 \left( \frac{B}{\phi \zeta_e} \right) \right] \quad \zeta_e < 1 \quad (40a)$$

$$= (\zeta_e - \frac{1}{2}) \left[ 1 + 0 \left( \frac{B}{\phi} \right) \right] \quad \zeta_e > 1 \quad (40b)$$

$$\frac{1}{\phi} \frac{E_e}{\epsilon H_{2,0}} = \zeta_e (3 - 2\zeta_e)^{1/2} \left\{ 1 - \frac{8(3 - 2\zeta_e)}{7(2 - \zeta_e)^3} \left[ 1 - \frac{7\zeta_e}{8} + \frac{7\zeta_e^2}{36} \right] \right\} \quad \zeta_e < 1 \quad (41a)$$

$$= 1 - \frac{8\zeta_e^3 - 12\zeta_e^2 + 6\zeta_e - (19/21)}{3(2\zeta_e - 1)^3} \quad \zeta_e > 1 \quad (41b)$$

$$\frac{t_p E}{\varepsilon H_{2,0}} = \zeta^2 (3 - 2\zeta_e) \left[ \frac{3 - 2\zeta}{3 - 2\zeta_e} - \frac{\zeta^4 (2 - \zeta)^2}{\zeta_e^4 (2 - \zeta_e)^2} \right] \quad \zeta_e < 1 \quad (42a)$$

$$= \zeta^2 (3 - 2\zeta) - \frac{\zeta^6 (2 - \zeta)^2}{(2\zeta_e - 1)^2} \quad \zeta < 1, \zeta_e > 1 \quad (42b)$$

$$= 1 - \frac{(2\zeta - 1)^2}{(2\zeta_e - 1)^2} \quad \zeta > 1, \zeta_e > 1 \quad (42c)$$

For  $\zeta_e < 1$ , the output energy  $E_e/\varepsilon H_{2,0}$  is proportional to  $\zeta_e (3 - 2\zeta_e)^{1/2}$  and a function of  $\zeta_e$  which varies from  $40/63$  to  $4/7$  as  $\zeta_e$  varies from 1 to 0. Hence, the output energy is essentially proportional to the F-atoms created by the flash lamp during the lasing process.

#### E. POWER LAW VARIATION OF $F_B$

A power law variation of  $F_B$  with time is now considered (Fig. 3). The objective is to determine the effect of the power law exponent on laser performance.

##### 1. ARBITRARY INITIATION LEVEL

We assume that F-atoms are created by a discharge according to the power law

$$F_B = at^n/n \quad n > 0 \quad (43a)$$

where  $a$  and  $n$  are known constants. The discharge is assumed to continue until lasing is complete. Thus, for the present case,  $t_B = t_e$ . These quantities are deduced in the course of the solution. The parameters  $\phi^2$  and  $t_p$  are functions of

$$F_{B,T} = a t_e^n/n \quad (43b)$$

which is known when  $t_e$  is determined.

In terms of  $\zeta$ , the regime of interest is  $\zeta < 1$  and  $\zeta_e = 1$ . It follows that for  $\zeta < 1$ ,

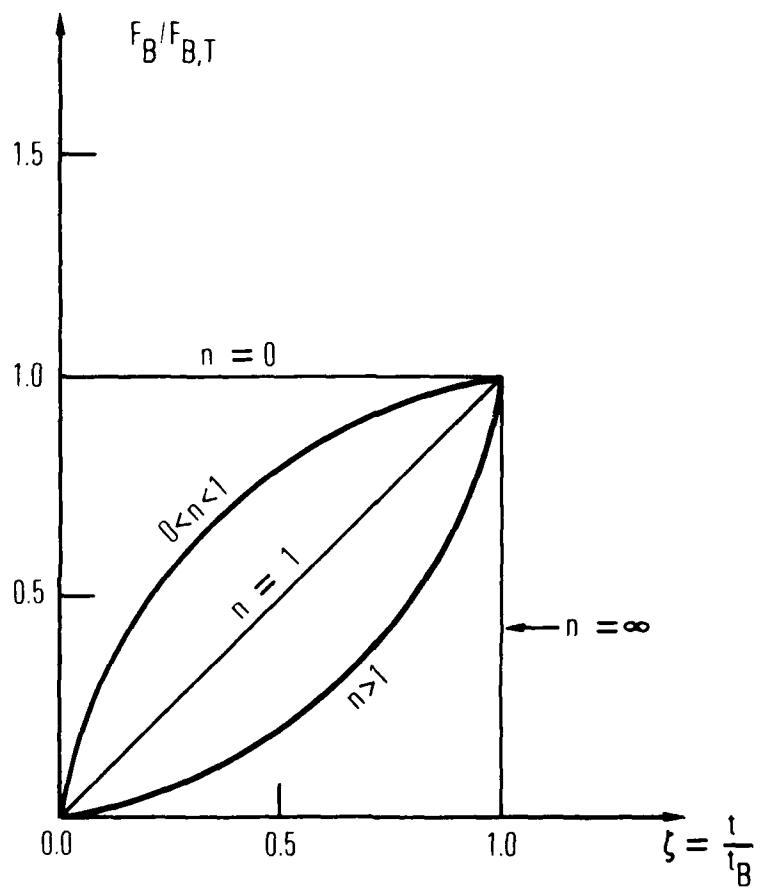


Fig. 3. F-Atom Concentration Associated with Power Law Initiation,  $F_B/F_{B,T} = \zeta^n$

$$F_B/F_{B,T} = \zeta^n \quad (44a)$$

$$\frac{1}{2} \frac{t_p}{t_B} \frac{HF}{H_{2,0}} = \frac{\zeta^n + 1}{n + 1} \quad (44b)$$

$$\frac{t_p \cdot E}{\epsilon H_{2,0}} = \zeta^n - \left( \frac{t_B}{\phi t_p} \right)^2 \frac{\zeta^n + 1}{n + 1} \left( \frac{\zeta^n + 1}{n + 1} + \frac{B t_p}{t_B} \right) \quad (44c)$$

The effect of  $n$  on  $F_B$  is illustrated in Fig. 3. The case  $n \rightarrow 0$  corresponds to a discharge where all the F-atoms are generated at  $t = 0$  (i.e.,  $t_B/t_e \ll 1$ ). The case  $n = 1$  corresponds to a uniform electron-beam discharge with  $t_B/t_e = 1$ . The case  $n \rightarrow \infty$  corresponds to the situation where most of the F-atoms are created at the end of the laser pulse. Since  $\zeta_e = 1$ , for the present case, the quantities  $t_e$  and  $t_B$  are interchangeable and are obtained from Eq. (44c). Thus

$$\frac{1}{n + 1} \frac{t_e}{\phi t_p} = \frac{1}{2} \left\{ [4 + (\frac{B}{\phi})^2]^{1/2} - \frac{B}{\phi} \right\} \quad (45a)$$

Integration of Eq. (44c), and evaluation at  $\zeta_e = 1$  provides the laser output power

$$(n + 1) \frac{t_p}{t_e} \frac{E_e}{\epsilon H_{2,0}} = 1 - \left( \frac{t_e}{\phi t_p} \right)^2 \left[ \frac{1}{(2n + 3)(n + 1)} + \frac{B t_p / t_e}{n + 2} \right] \quad (45b)$$

where  $\phi t_p / t_e$  is obtained from Eq. (45a). The solution depends only on  $B/\phi$ . The limits of weak and strong initiation are as follows.

## 2. WEAK INITIATION $(\phi/B)^2 \ll 1$

$$\frac{1}{n + 1} \frac{B}{\phi^2} \frac{t_e}{t_p} = (n + 2) \frac{B}{\phi^2} \frac{E_e}{\epsilon H_{2,0}} = 1 + O(\frac{\phi}{B})^2 \quad (46a)$$

$$t_p \cdot E / (\epsilon H_{2,0}) = \zeta^n (1 - \zeta) \quad (46b)$$

From Eq. (46a)

$$t_e = (n + 1) / \left( \sum_i k_{cd}^{M_i} M_i \right) \quad (46c)$$

Hence,  $t_e$  is known and all other laser properties can be deduced from Eqs. (43) and (46).

### 3. STRONG INITIATION ( $B/\phi \ll 1$ )

$$\frac{1}{n+1} \frac{t_e}{\phi t_p} = \frac{2n+3}{n+2} \frac{1}{\phi} \frac{E_e}{\epsilon H_{2,0}} = 1 + O(\frac{B}{\phi}) \quad (47a)$$

$$t_p \frac{E_e}{\epsilon H_{2,0}} = \zeta^n (1 - \zeta^{n+2}) \quad (47b)$$

From Eq. (47a)

$$\frac{k_{cd}^{HF} t_e E_e}{\epsilon} = \frac{(n+1)(n+2)}{2(2n+3)} \quad (47c)$$

Laser properties are deduced from Eqs. (43) and (47).

### III. DISCUSSION

Results deduced in the previous section are summarized in Table 1 and are discussed herein. In particular, the physical significance of the parameters associated with weak and strong initiation is noted. The effect of initiation pulse length on laser performance, the region of validity of the present theory, and improved scaling laws are then discussed.

#### A. PHYSICAL SIGNIFICANCE OF PARAMETERS

Typical values of parameters are listed in Appendix A. If kinetic rate coefficients are based on values at  $T = 300$  K, the quantities  $B$  and  $\phi^2$  become

$$B \equiv \frac{\sum_i k_{cd}^{M_i} M_i}{2 H_{2,0} k_{cd}^{HF}} \approx \frac{1}{2} \frac{k_{cd}^{H_2}}{k_{cd}^{HF}} = 0.89 \times 10^{-2} \quad (48a)$$

$$\phi^2 = 0.50 \frac{(F_{B,T}/F_{2,0})(F_2/H_2)_0^2}{1 + 0.094 (F_2/H_2)_0} \quad (48b)$$

The approximation in Eq. (48a) is generally correct to about 10% (Appendix A) and is based on the assumption that  $H_2$  is the major collisional deactivator for species other than HF. In the latter case,  $B$  is independent of stoichiometry and is of the order of  $10^{-2}$ . The parameter  $\phi^2$  is dependent upon stoichiometry.

Let  $(HF)_e$  denote the amount of HF generated during the laser pulse. This quantity may be approximated from Eq. (17) by

$$\frac{(HF)_e}{2 H_{2,0}} = \int_0^{t_e} \frac{F_B}{F_{B,T}} \frac{dt}{t_p} \sim \frac{t_e}{t_p} \frac{F_{B,e}}{F_{B,T}} \quad (49)$$

where the symbol  $\sim$  denotes order of magnitude. Eq. (49) is used to characterize the role of HF as a collisional deactivation partner for the weak and strong initiation cases.

Table 1. Summary of Results for Pulsed Chemical Laser Performance

Initiation regime	Parameter	Electron Beam			Flash Lamp			Power Law	
		$\zeta_e^{-1} = 0$	$\zeta_e^{-1} = 1$	$\zeta_e^{-1} = \infty$	$\zeta_e^{-1} = 0$	$\zeta_e^{-1} = 1$	$\zeta_e^{-1} = \infty$	$\zeta_e^{-1} = 0$	$\zeta_e^{-1} = 1$
Weak	$\frac{t_e}{(\sum_i M_i k_{cd})^{-1}}$	1	2	2	1	2	3	n	n+1
Weak	$2 \frac{B}{\phi^2} \frac{E_e}{\epsilon H_{2,0}}$	1	$\frac{2}{3}$	$\frac{2}{3} \zeta_e$	1	$\frac{4}{5}$	$\frac{3}{2} \zeta_e^2$	$\frac{2}{n+2}$	
Strong	$(2 H_{2,0} k_{cd}^H \phi) t_e$	1	2	$\frac{2}{\zeta_e^{1/2}}$	1	2	$\frac{3^{1/2}}{\zeta_e}$		n+1
Strong	$\frac{3}{24} \frac{E_e}{\epsilon H_{2,0}}$	1	$\frac{9}{10}$	$\frac{9}{10} \zeta_e^{1/2}$	1	$\frac{20}{21}$	$\frac{18}{7} \zeta_e$		
Strong	$3 k_{cd}^H \frac{E_e t_e}{\epsilon}$	1	$\frac{9}{5}$	$\frac{9}{5}$	1	$\frac{40}{21}$	$\frac{18}{7}$	$\frac{3(n+2)}{2(2n+3)} (n+1)$	

## 1. WEAK INITIATION $(\phi/B)^2 \ll 1$

In the weak initiation regime, the laser pulse length is (e.g., Table 1)

$$t_e = \left( \sum_i k_{cd}^{M_i} M_i \right)^{-1} f_1(\zeta_e) \quad (50)$$

where  $f_1(\zeta_e)$  denotes a function of  $\zeta_e$ . Thus, the laser pulse length is of the order of the time for collisional deactivation of  $HF_u$  by species other than HF (generally,  $H_2$ ). Substitution of Eqs. (49) and (50) into Eqs. (7) and (18) indicate, for  $F_{B,T}/F_{B,e} = 0$  (1),

$$\left( \frac{\phi}{B} \right)^2 \sim \frac{k_{cd}^{HF} (HF)_e}{\sum_i k_{cd}^{M_i} M_i} \quad (51)$$

Thus  $(\phi/B)^2$ , which is assumed small in the present limit, equals the ratio of the rate of deactivation of  $HF_u$  by HF to the rate of deactivation of  $HF_u$  by all other species. The assumption that  $(\phi/B)^2$  is small indicates that the amount of HF produced during the lasing process is so small that its contribution to the collisional deactivation of  $HF_u$  is negligible. Eq. (50) is a consequence of this situation. It also follows (from Table 1)

$$\frac{E_e}{\epsilon H_{2,0}} = \frac{\phi^2}{B} f_2(\zeta_e) \quad (52)$$

where  $f_2(\zeta_e)$  denotes a function of  $\zeta_e$ . Hence, the chemical efficiency is very low. Note that  $E_e/\epsilon H_{2,0}$  has a linear dependent on  $F_{B,T}/F_{B,e}$  as noted in Reference 8.

## 2. STRONG INITIATION $(B/\phi) \ll 1$

In the strong initiation regime,  $t_e \sim \phi t_p$  (Table 1) and for  $F_{B,T}/F_{B,e} = 0(1)$ ,

$$\frac{B}{\phi} \sim \frac{\sum_i k_{cd}^{M_i} M_i}{k_{cd}^{HF} (HF)_e} \quad (53)$$

Thus,  $B/\phi$  is the ratio of the rate of collisional deactivation of  $\text{HF}_u$  by species  $M_1$  to the rate of collisional deactivation by HF. The assumption  $B/\phi \ll 1$  implies that HF is the major collisional deactivation partner. In this regime

$$E_e/(\epsilon H_{2,0}) = \phi f_3(\zeta_e) \quad (54a)$$

$$H_{2,0} t_e = (k_{cd}^{\text{HF}} \phi)^{-1} f_4(\zeta_e) \quad (54b)$$

$$E_e t_e / \epsilon = (k_{cd}^{\text{HF}})^{-1} f_5(\zeta_e) \quad (54c)$$

Eq. (54a) indicates that  $E_e/(\epsilon H_{2,0})$  is proportional to  $(F_{B,T}/F_{2,0})^{1/2}$ . The square root dependence may be compared with the linear dependence in Eq. (52). Note that Eq. (54a) does not depend on  $H_{2,0}$  when  $0.094(F_2/H_2)_0 \ll 1$ . Equation (54c) indicates that the product  $E_e t_e$  is independent of stoichiometry.

#### B. INITIATOR PULSE LENGTH EFFECT

The effect on laser performance of the initiator pulse length parameter  $\zeta_e^{-1}$  and the power law exponent,  $n$ , can be readily deduced from Table 1.

The dependent variables in Table 1 are normalized to equal one in the limit of a short initiator pulse ( $\zeta_e^{-1} = 0$  or  $n = 0$ ). In the weak initiation regime, an increase of  $\zeta_e^{-1}$  from 0 to 1.0 results in a decrement of laser output energy of 20 and 33-1/3% for the flash lamp and electron-beam initiations, respectively. The results in the strong initiation regime are of major interest. As  $\zeta_e^{-1}$  increases from 0 to 1.0, the laser output energy is reduced by a factor of only 5 and 10% for the flash lamp and the electron-beam, respectively. For the power law initiator, the energy decrement in the strong initiation regime is only 25% in the limit  $n \rightarrow \infty$ . (In the latter limit the F-atoms are created essentially at the end of the laser pulse.) The laser pulse length,  $t_e$ , is increased by a factor of two, as  $\zeta_e^{-1}$  increases from 0 to 1, for both the flash lamp and the electron beam. The same factor of two applies for weak as well as strong initiation. The agreement between the flash lamp and

electron-beam cases appears to be fortuitous, since the laser pulse length has a value  $n + 1$  for power law initiation. Hence, the pulse length is increased as the F-atom generation is deferred to the end of the laser pulse.

The relative insensitivity of saturated laser output energy to initiator beam duration, in the strong initiation region, can be readily explained. The laser pulse terminates when the rate of creation of  $HF_u$  equals its rate of collisional deactivation. In the strong initiation regime, HF is the major collisional deactivator. Hence, lasing does not terminate until sufficient HF has been created. But saturated laser output energy also depends on the amount of HF generated during the laser pulse. Hence, the output energy is relatively insensitive to the initiator discharge pulse shape (e.g.,  $n$ ). The length of the laser pulse, however, is dependent on  $n$  since the laser pulse will not terminate until sufficient HF has been created. The sensitivity of laser output energy to initiator pulse length is expected to increase with a decrease in laser saturation.

The results for  $\zeta_e^{-1} = \infty$ , in Table 1, reflect the earlier observation e.g., Eq. (24a), that laser performance depends on F-atom generation during the actual laser pulse. Alternate parameters (e.g., replacement of  $F_{B,T}$  by  $F_{B,e}$ ) should be used when  $\zeta_e^{-1}$  is considerably larger than one.

The assumption  $t_e/t_p \ll 1$  has been employed. As shown in Table 1, the solution for a power law initiator is limited to values of  $n$  in the range  $(n + 1) \ll \phi^2/B$  and  $(n + 1) \ll 1/\phi$  for the weak and strong initiation regimes, respectively.

An experimental study of the effect of electron-beam duration on pulsed chemical laser performance was reported in Reference 9. Experimental results for a fixed value of  $F_{B,T}$  (e.g.,  $3.5 \mu\text{Cb}/\text{cm}^2$ ) showed that as the electron-beam duration was increased from 0.07 to 0.6  $\mu\text{sec}$ , the laser output energy remained the same, and the laser pulse length, FWHM, increased from 0.5 to 0.8  $\mu\text{sec}$ . If it is assumed that  $t_B$  is approximately equal to two times FWHM, the experimental results correspond to  $\zeta_e^{-1} = 0.07$  and 0.38, respectively. The present analytical model predicts no change in output energy, Eq. (29), and a 20% increase in laser pulse length, Eq. (28). The energy prediction is in exact

agreement with the experiment. The laser pulse length prediction, however, is only in fair agreement with the 60% increase indicated by the experiment.

### C. LASER PULSE SHAPE

The variation of laser output intensity with time in the weak and strong initiation regimes is illustrated in Figs. 4 and 5. When  $\zeta_e^{-1} = 0$ , the peak intensity occurs at  $\zeta/\zeta_e = 0$ . With increase in  $\zeta_e^{-1}$ , the peak intensity occurs at increased values of  $\zeta/\zeta_e$ . Let  $\zeta_m$  denote the location of the peak intensity. For an electron-beam initiator, the value of  $\zeta_m$  is obtained from:

#### 1. WEAK INITIATION

$$\zeta_m/\zeta_e = \zeta_e^{-1} \quad \zeta_e^{-1} < 2/3 \quad (55a)$$

$$= (2\zeta_e - 1)/(2\zeta_e) \quad (2/3) < \zeta_e^{-1} < 1 \quad (55b)$$

#### 2. STRONG INITIATION

$$\frac{\zeta_m}{\zeta_e} = \zeta_e^{-1} \quad \zeta_e^{-1} < 2/3 \quad (56a)$$

$$= \left[ \frac{(2\zeta_e - 1)}{2} \right]^{2/3} \zeta_e^{-1} \quad \frac{2}{3} < \zeta_e^{-1} < 1 \quad (56b)$$

Thus, for  $\zeta_e^{-1} < 2/3$ , the peak intensity occurs at the end of initiator pulse (i.e.,  $\zeta_m = 1$ ). For  $2/3 < \zeta_e^{-1} < 1$ , the peak intensity occurs in the range  $2/3 > \zeta_m/\zeta_e > 1/2$  and  $2/3 > \zeta_m/\zeta_e > 0.630$  for weak and strong electron-beam initiation, respectively.

Experimental results for the variation of laser intensity with time are given in Fig. 6 for the strong electron-beam initiation case.<sup>9</sup> The experimental data indicate  $\zeta_m/\zeta_e = 0.25, 0.6$  for  $\zeta_e^{-1} = 0.25, 1.0$ , respectively.

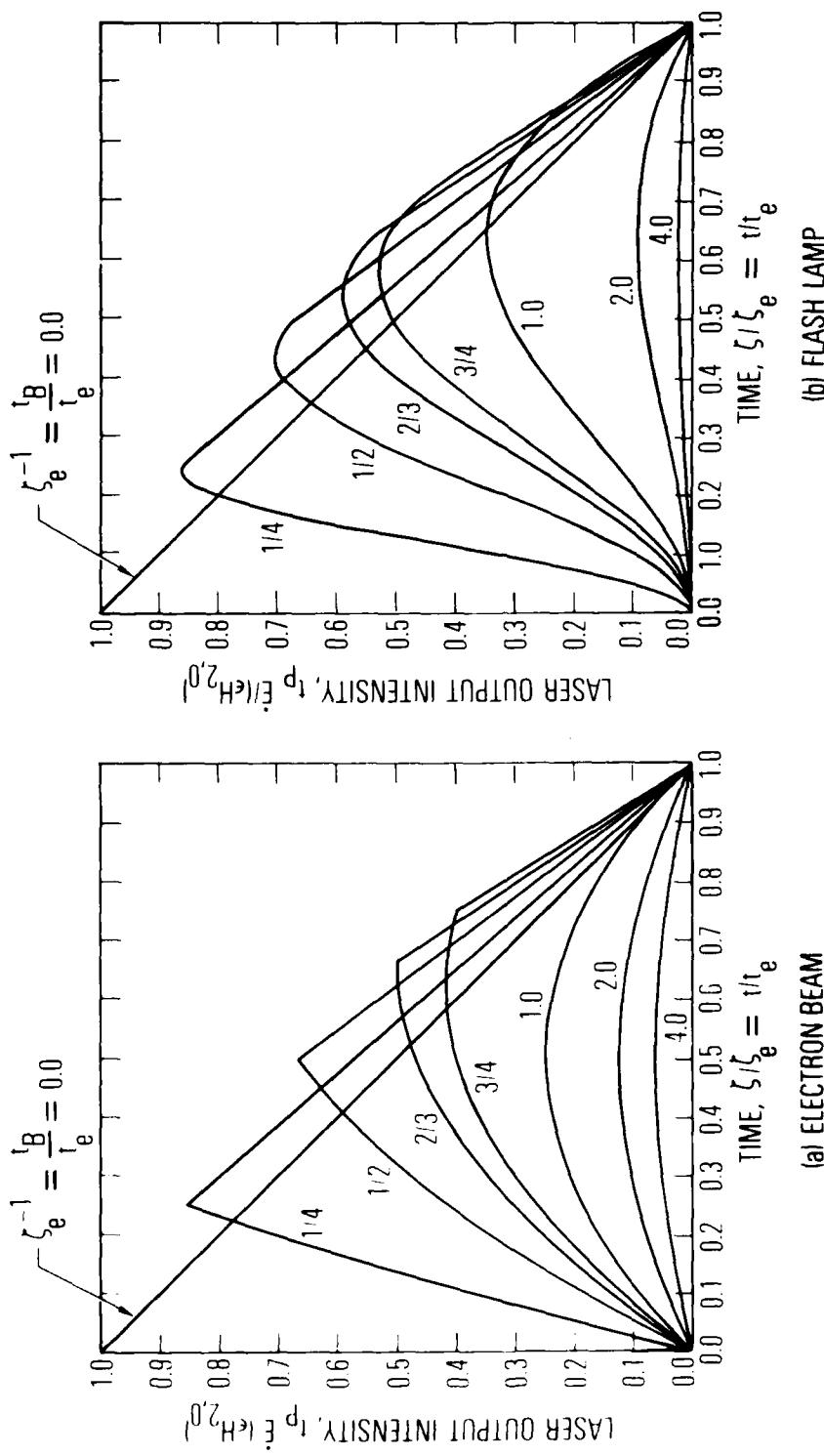


Fig. 4. Laser output Intensity Variation for Weak Initiation,  $(\phi/B)^2 \ll 1$ .  
 (a) Electron beam and (b) flash lamp

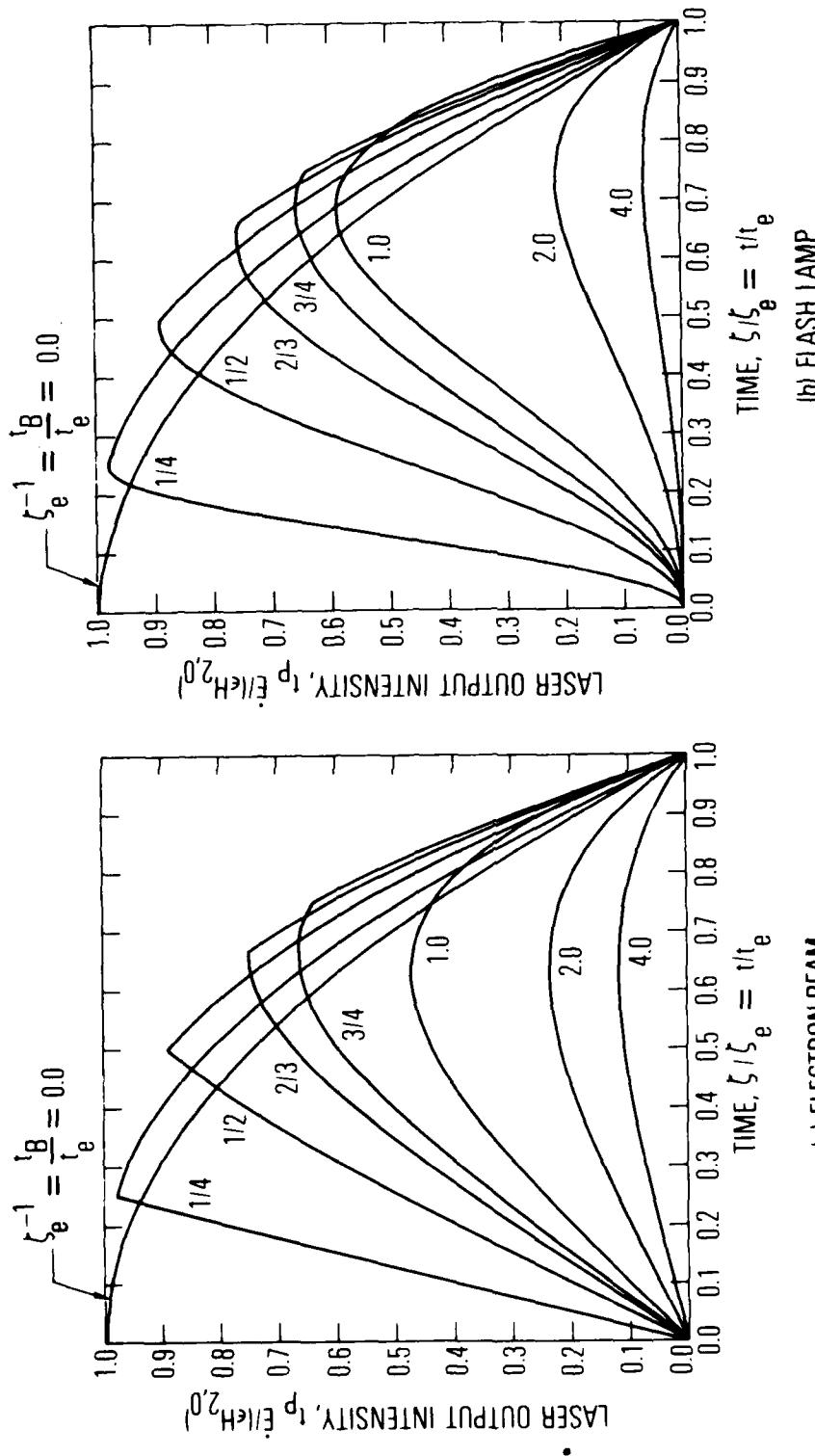


Fig. 5. Laser Output Intensity Variation for Strong Initiation,  $(R/\Phi) \ll 1$ .  
 (a) Electron beam and (b) flash lamp

These results are in exact agreement with the analytic prediction, Eq. (56). Further comparisons are desirable for  $2/3 < \zeta_e^{-1} < 1$ , however, because of the nonanalytic form of the pulse shape in Fig. 6(b).

The variation of laser intensity with time, for a flash-initiated laser in the strong initiation regime is given in Fig. 7.<sup>3</sup> The experimental data indicate  $\zeta_m/\zeta_e = 0.44$  and  $\zeta_e^{-1} = 0.65$ . The experimental variation of intensity with time, however, appears to correlate more closely with the results for  $\zeta_e^{-1} = 1/4, 1/2$  in Fig. 7. Further comparisons are also needed for the flash-initiated case.

#### D. REGION OF VALIDITY

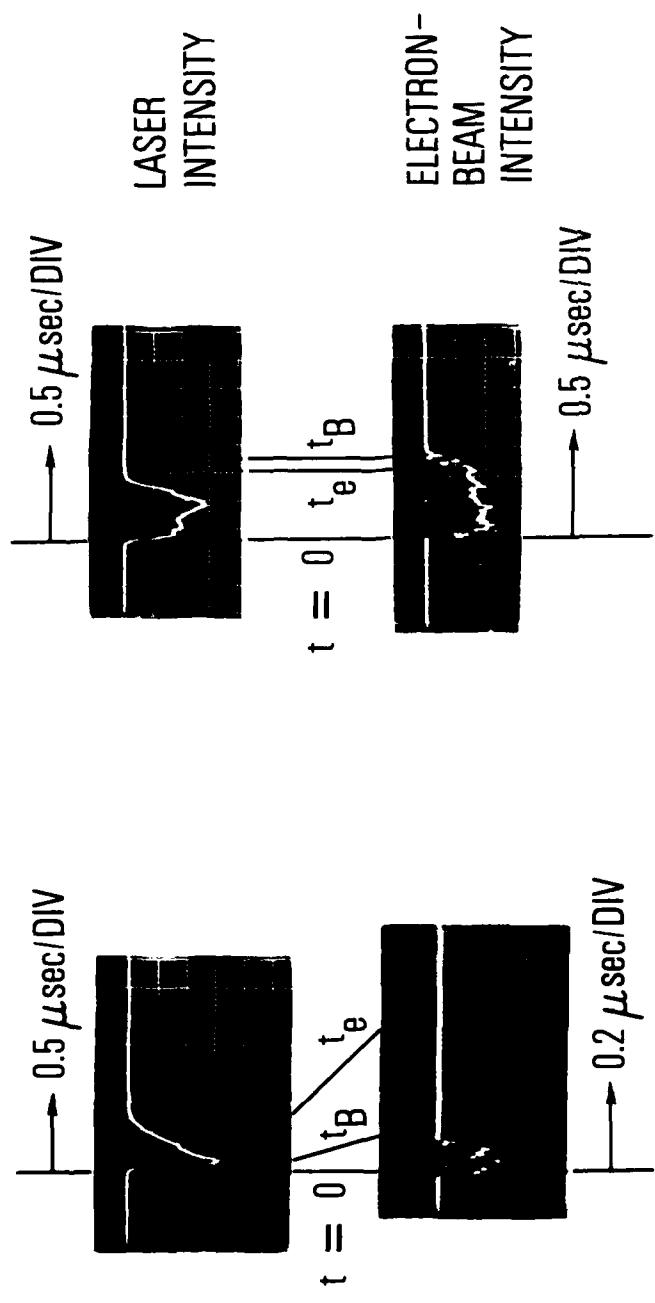
The present theory is self-consistent provided the inequalities in Eq. (1) are satisfied. Consideration of Eqs. (14b) and (18b) indicates that  $t_t/t_p$  is of the order of  $\phi^2$ . Moreover,  $t_e/t_p$  is of order  $\phi^2/B$  and  $\phi$  in the weak and strong initiation regimes, respectively [e.g., Eqs. (24), (28), (37), (40)]. Hence, the present theory is self-consistent when  $\phi \ll 1$ . The weak and strong initiation regimes are thus bounded by

$$\begin{array}{ll} \phi^2 \ll B^2 & \text{weak} \\ B \ll \phi \ll 1 & \text{strong} \end{array}$$

An improved treatment of the chain reaction is needed when  $\phi$  is of the order of one.

#### E. IMPROVED SCALING LAWS

The numerical coefficients that appear in the present scaling laws are expected to be in error because of the simplified two-level model. Improved coefficients were obtained in Reference 8, for the case  $\zeta_e^{-1} = 0$ , by comparing the theoretical expressions with experimental data. The improved scaling laws noted in Reference 8 are now generalized to include the effect of  $\zeta_e^{-1} \neq 0$  as deduced herein. The case of an electron-beam initiated HF laser, strong initiation region, and  $0 < \zeta_e^{-1} < 1$  is considered. From Eqs. (28) and (29) and Eq. (34) of Reference 8, and for  $0.02 < \psi < 1$  (strong initiation),



(a) SHORT ELECTRON-BEAM PULSE

$$\frac{t_B}{t_e} = \frac{1}{4} ; \frac{t_m}{t_e} = \frac{1}{4}$$

(b) LONG ELECTRON-BEAM PULSE

$$\frac{t_B}{t_e} = 1.0 ; \frac{t_m}{t_e} = 0.6$$

Fig. 6. Effect of Electron-Beam Duration on Variation of Laser Output Intensity with Time in Strong Initiation Regime. (a) Short electron-beam pulse and (b) long electron-beam pulse (data from Reference 9)

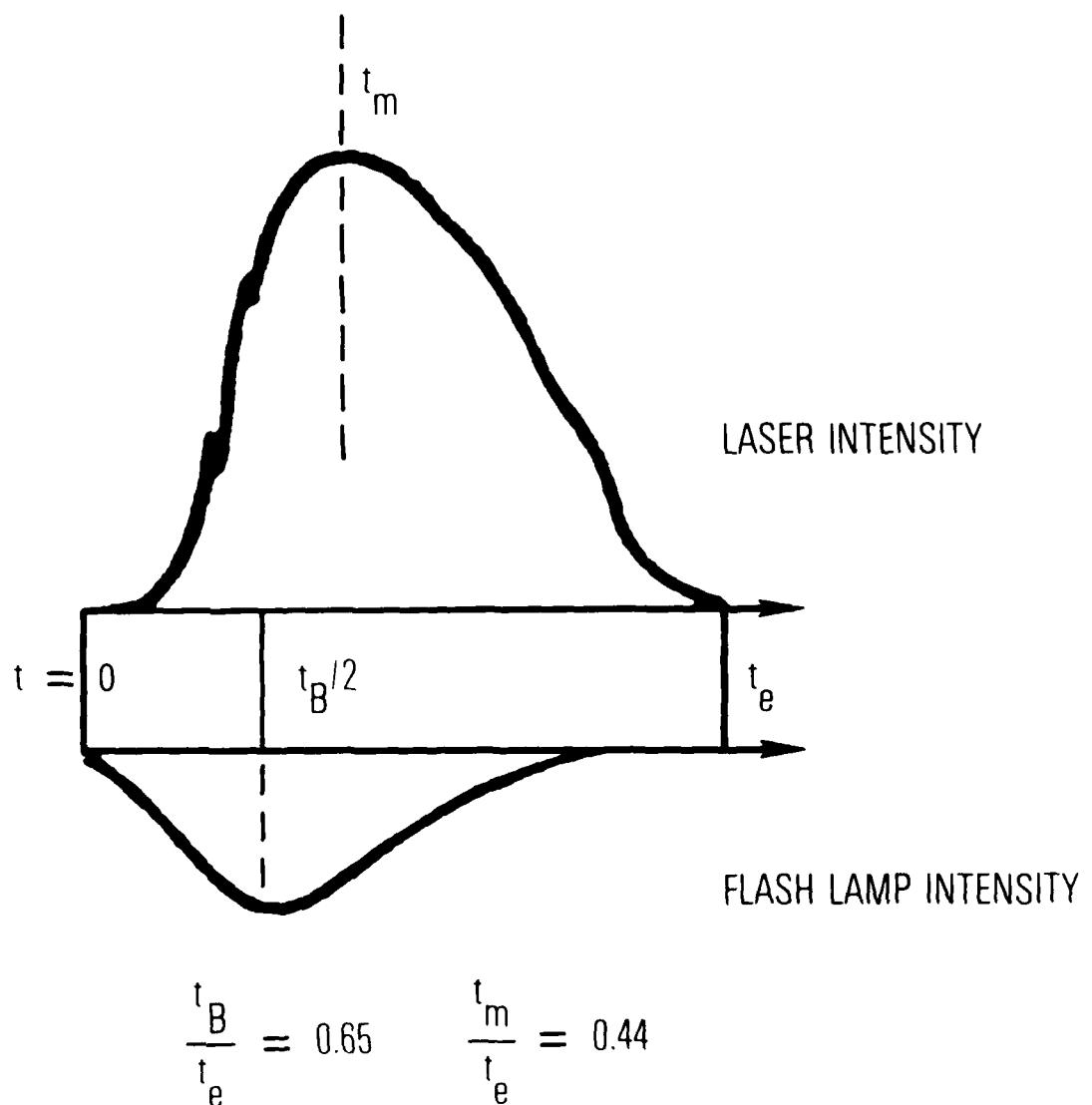


Fig. 7. Variation of Laser Intensity with Time for Flash-Initiated Laser in Strong Initiation Regime (data from Reference 3)

$$\frac{E_e}{p_{H_2,0}} = 4.6 \psi \left( 1 - \frac{J}{10(2\zeta_e - 1)^3} \right) \frac{1}{cm^3 atm} \quad (57a)$$

$$p_{H_2,0} t_e = 2.4 \times 10^{-8} / \left[ \psi \left( 1 - \frac{\zeta_e^{-1}}{2} \right) \right] atm sec \quad (57b)$$

$$E_e t_e = 0.11 \times 10^{-6} \left( 1 - \frac{1}{10(2\zeta_e - 1)^3} \right) / \left( 1 - \frac{\zeta_e^{-1}}{2} \right) \frac{J sec}{cm^3} \quad (57c)$$

where  $p_{H_2,0}$  is the partial pressure of  $H_2$  in atmospheres and

$$\psi = \left( \frac{F_{B,T}}{F_{2,0}} \right)^{1/2} \left( \frac{F_2}{H_2} \right)_0 \left[ 1 + 0.094 \left( \frac{F_2}{H_2} \right)_0 \right]^{-1/2} \quad (57d)$$

Eq. (57), evaluated at  $\zeta_e^{-1} = 1$ , applies for  $\zeta_e^{-1} > 1$  provided  $F_{B,T}$  in Eq. (57d) is replaced by  $F_{B,e}$ .

#### IV. CONCLUDING REMARKS

The present study is a generalization of Reference 8 and effects of  $\zeta_e^{-1} \neq 0$  are investigated. However, the deactivation model in Reference 8 differs somewhat from the model used herein, as is discussed in Appendix B.

The results of the present study are summarized in Table 1. The major conclusion is that, in the strong initiation regime, output energy is relatively insensitive to  $\zeta_e^{-1}$  in the range  $0 < \zeta_e^{-1} < 1$ .

The replacement of the parameter  $\phi^2$  by  $\phi^2 F_{B,e}/F_{B,T}$  would be physically meaningful, for  $\zeta_e^{-1} > 1$ , since the latter is based on the F-atoms created during the lasing process  $F_{B,e}$ , whereas the former is based on the total F-atom production  $F_{B,T}$ . The parameter  $\phi^2$  was used herein since it is a known system parameter. The quantity  $\phi^2 F_{B,e}/F_{B,T}$  might be useful, however, for correlating experimental data.

#### REFERENCES

1. H. L. Chen, R. L. Taylor, J. Wilson, P. Lewis, and W. Fyfe, *J. Chem. Phys.* 61, 306 (1974).
2. D. B. Nichols, R. B. Hall, and J. D. McClure, *J. Appl. Phys.* 47, 4026 (1976).
3. A. S. Bashkin, N. P. Vagin, O. R. Nazyrov, A. N. Oraevski, V. S. Pazyuk, O. E. Porodinkov, and N. N. Yuryshev, *Sov. J. Quantum Electron.* 10, 1821 (1980).
4. V. F. Zharov, V. K. Malinovskii, Yu. S. Neganov, and G. M. Chumak, *JETP Lett.* 16, 154 (1972).
5. J. A. Mangano, R. L. Limpaecher, J. D. Daugherty, and F. Russell, *Appl. Phys. Lett.*, 27, 293 (1975).
6. R. A. Gerber and E. L. Patterson, *J. Appl. Phys.* 47, 3524 (1976).
7. R. Hofland, A. Ching, M. L. Lundquist, and J. S. Whittier, *IEEE J. Quantum Electron.* QE-10, 781 (1974); *J. Appl. Phys.* 47, 4543 (1976).
8. H. Mirels, R. Hofland, and J. S. Whittier, *J. Appl. Phys.* 50, 6660 (1979).
9. S. T. Amimoto, J. S. Whittier, M. L. Lundquist, F. G. Ronkowski, and R. Hofland, Jr., "Pulsed Chemical Laser with Variable Pulse-Length E-Beam Initiation and Magnetic Confinement," The Aerospace Corporation, submitted to *Appl. Opt.*

## APPENDIX A

### TYPICAL VALUES OF PARAMETERS

RATE COEFFICIENTS<sup>8</sup> (T = 300 K, k = cm<sup>3</sup>/mole-sec)

$$k_c = 1.09 \times 10^{13} \quad k_{cd}^F = 1.73 \times 10^{11}$$

$$k_h = 1.02 \times 10^{12} \quad k_{cd}^H = 1.39 \times 10^{11}$$

(A-1)

$$k_{cd}^{HF} = 1.01 \times 10^{12} \quad k_{cd}^{He} = 3.74 \times 10^6$$

$$k_{cd}^{H_2} = 1.80 \times 10^{10} \quad k_{cd}^{F_2} = 1.87 \times 10^6$$

INITIAL CONDITIONS (strong initiation case)

$$H_{2,0}/F_{2,0}/H_e = 1/2/7$$

$$F_{B,T}/F_{2,0} = 10^{-2}$$

(A-2)

$$p = 1 \text{ atm}$$

$$T = 300 \text{ K}$$

PARAMETERS From Eqs. (A-1) and (A-2)

$$R = 0.158$$

$$B = \frac{k_{cd}^{H_2} H_{2,0} + k_{cd}^{H_e} H_e + k_{cd}^{F_2} F_{2,0} + k_{cd}^F (RF_{B,T}) + k_{cd}^H [(1-R)F_{B,T}]}{2 H_{2,0} k_{cd}^{HF}}$$

$$= 1.03 \times 10^{-2}$$

$$\frac{1}{2} \frac{k_{cd}^{H_2}}{k_{cd}^{HF}} = 0.89 \times 10^{-2}$$

$$\phi = 1.30 \times 10^{-1}$$

$$B/\phi = 0.792 \times 10^{-1}$$

$$t_t/t_p = 2.66 \times 10^{-3}$$

$$t_e/t_p = 1.30 \times 10^{-1} \quad (\zeta_e^{-1} = 0)$$

## APPENDIX B

### COMPARISON WITH REFERENCE 8

Pulsed chemical-laser performance is deduced in Reference 8 for  $\zeta_e^{-1} = 0$  and arbitrary degrees of saturation. The present study is a generalization of Reference 8 and treats a saturated pulsed chemical laser for arbitrary values of  $\zeta_e$ . However, the rate of collisional deactivation by HF is treated somewhat differently in each study. The deactivation term in Reference 8, Eq. (23b), is  $k_{cd}^{HF} (HF)_e HF_u$  which employs  $(HF)_e$  to characterize the concentration of HF collision partners. This approximation was introduced to provide simple closed-form solutions in the case of arbitrary degrees of saturation. The corresponding term in the present study, Eq. (4), is  $k_{cd}^{HF} HF HF_u$ , which is exact but permits closed-form solutions only in the case of a saturated laser. The functional form of the present theory, in the limit  $\zeta_e^{-1} = 0$ , agrees with Reference 8. Certain numerical coefficients differ, however, in the strong initiation limit. In view of the simplified two-level model used herein, and in Reference 8, the functional form of the solutions is the main concern. Numerical coefficients should be adjusted to conform with experimental data as in Eq. (57).

## SYMBOLS

B	collisional deactivation rate ratio parameter, Eq. (7c)
E	laser output energy per unit volume up to time $t$ , Eq. (7b)
$F_B$	F-atoms per unit volume generated by initiator beam, Eq. (9)
$f_n(\zeta_e)$	function of $\zeta_e$ , $n = 1, 2, \dots 5$
g	small signal gain, Eq. (2b)
I	local photon flux
k	rate coefficient, $\text{cm}^3/\text{mole-sec}$
$k_{cd}^{\text{HF}}, k_{cd}^{M_i}$	rate coefficients for collisional deactivation, Eq. (4)
$M_i$	species other than HF
n	power law exponent, Eq. (43a)
R	chain reaction parameter, Eq. (15b)
t	time, sec
$t_B, t_e$	initiator pulse time and laser pulse time, respectively, Eqs. (9c) and (23a)
$t_e, t_p$	initial transient and overall time for chain reaction, Eqs. (14b) and (18a)
$\epsilon$	energy per mole of photons, 44.0 KJ/mole for $\lambda = 2.72 \mu\text{m}$
$\zeta, \zeta_e$	normalized time; $t/t_B, t_e/t_B$
$\sigma$	stimulated emission cross section, Eq. (2b)
$\phi$	initiation strength parameter, Eq. (18b)
$\downarrow$	alternate initiation parameter, Eq. (57d)

### SUBSCRIPTS

B	pertaining to initiator beam
c	cold reaction, Eq. (8a)

cd        collisional deactivation  
e        end of laser pulse  
h        hot reaction, Eq. (8b)  
l        lower vibrational level, Eq. (2a)  
m        time corresponding to peak value of  $E$  , Eq. (55)  
p        pumping reaction, Eq. (3)  
u        upper vibrational level, Eq. (2a)  
T        total value, Eq. (9)

SUPERSCRIPT

( $\dot{}$ )         $d( )/dt$

## LABORATORY OPERATIONS

The Laboratory Operations of The Aerospace Corporation is conducting experimental and theoretical investigations necessary for the evaluation and application of scientific advances to new military space systems. Versatility and flexibility have been developed to a high degree by the laboratory personnel in dealing with the many problems encountered in the nation's rapidly developing space systems. Expertise in the latest scientific developments is vital to the accomplishment of tasks related to these problems. The laboratories that contribute to this research are:

Aerophysics Laboratory: Launch vehicle and reentry fluid mechanics, heat transfer and flight dynamics; chemical and electric propulsion, propellant chemistry, environmental hazards, trace detection; spacecraft structural mechanics, contamination, thermal and structural control; high temperature thermomechanics, gas kinetics and radiation; cw and pulsed laser development including chemical kinetics, spectroscopy, optical resonators, beam control, atmospheric propagation, laser effects and countermeasures.

Chemistry and Physics Laboratory: Atmospheric chemical reactions, atmospheric optics, light scattering, state-specific chemical reactions and radiation transport in rocket plumes, applied laser spectroscopy, laser chemistry, laser optoelectronics, solar cell physics, battery electrochemistry, space vacuum and radiation effects on materials, lubrication and surface phenomena, thermionic emission, photosensitive materials and detectors, atomic frequency standards, and environmental chemistry.

Electronics Research Laboratory: Microelectronics, GaAs low noise and power devices, semiconductor lasers, electromagnetic and optical propagation phenomena, quantum electronics, laser communications, lidar, and electro-optics; communication sciences, applied electronics, semiconductor crystal and device physics, radiometric imaging; millimeter wave, microwave technology, and RF systems research.

Information Sciences Research Office: Program verification, program translation, performance-sensitive system design, distributed architectures for spaceborne computers, fault-tolerant computer systems, artificial intelligence and microelectronics applications.

Materials Sciences Laboratory: Development of new materials: metal matrix composites, polymers, and new forms of carbon; nondestructive evaluation, component failure analysis and reliability; fracture mechanics and stress corrosion; analysis and evaluation of materials at cryogenic and elevated temperatures as well as in space and enemy-induced environments.

Space Sciences Laboratory: Magnetospheric, auroral and cosmic ray physics, wave-particle interactions, magnetospheric plasma waves; atmospheric and ionospheric physics, density and composition of the upper atmosphere, remote sensing using atmospheric radiation; solar physics, infrared astronomy, infrared signature analysis; effects of solar activity, magnetic storms and nuclear explosions on the earth's atmosphere, ionosphere and magnetosphere; effects of electromagnetic and particulate radiations on space systems; space instrumentation.

END

FILMED

2-86

DTIC